MACHINE SCIENCE

ISSN: 2227-6912
E-ISSN: 2790-0479

2024, № 1 Pages 4 – 10

INTEGRATED CHARACTERIZATION OF TECHNOLOGICAL SYSTEM COMPLIANCE

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Abstract: This paper introduces the concept of a comprehensive compliance characteristic for technological systems in metal-cutting operations. Within a full-factor model that accounts for all spatial constraints acting on a body under load, the elastic properties of deformable constraints are represented by a generalized compliance matrix. This matrix reflects not only the intrinsic elastic properties of a subsystem but also the setup parameters, including the position of the cutting force application point relative to the subsystem reference point. For a decomposed technological system, each subsystem is assigned its own compliance matrix. The complete compliance characteristic of the system is defined by a set of such matrices, the number of which depends on the number of supports in the setup. It is shown that, for practical implementation, it is sufficient to determine two basic matrices: the compliance matrix for translational displacements and the angular compliance matrix for rotational displacements. Each matrix element represents the displacement component along a specific coordinate direction caused by a unit force or moment applied along a given axis. An experimental method for determining static matrix compliance characteristics is proposed. The method is based on applying three linearly independent forces (or moments) and measuring the resulting translational and angular displacements. Solving the corresponding matrix equations yields all elements of the required compliance matrices. Approaches for dynamic and statistical evaluation are also considered. Modal analysis is identified as an effective tool for dynamic characterization, while a production-based method adapted to matrix representation enables operational assessment directly under cutting conditions on modern CNC machines.

Keywords: *Comprehensive compliance characteristic, technological subsystem, compliance matrix, angular compliance, static and dynamic stiffness, CNC diagnostics.*

Introduction.

In [1-12] it is shown that, when fully accounting for the entire set of constraints determining the position of a body in space under the action of applied forces—i.e., within a full-factor (complete) model—the elastic properties of deformable constraints are completely described by a matrix:

$$E = e - a_0 \xi a_0 \qquad (1)$$

where, e – the compliance matrix; a_{00} – the matrix defining the coordinate vector of the point of force application $\overline{P}$; ξ – the angular compliance matrix. The introduced matrix E characterizes the elastic deformation properties of the entire set of constraints, limiting the motion of the body in space under the action of the applied load; therefore, it makes sense to call the matrix E the comprehensive compliance characteristic of the body (subsystem). With respect to a technological system, its own matrix E_i is introduced for each of the subsystems into which the technological system is decomposed. Thus, for single-support setups, the compliance of the technological system is fully described by a set of two E-matrices: E_1 and E_2. For two-support setups, the complete compliance characteristic of the technological system contains three E-matrices: E_0, E_1, E_2. Accordingly, for a multi-support setup (consisting of *n* supports), the complete compliance characteristic contains an n + 1 E- matrix set: E_0 and $\{E_i\}_n$.

The introduced comprehensive compliance characteristic of a technological subsystem (1) includes, in addition to the subsystem's own elastic properties, also the parameters of the setup for which this compliance is considered. The matrix a_0 characterizes the vector connecting the reference point of the subsystem and the point of application of the cutting forces.

As a reference point of the subsystem, it is reasonable to take the point of the subsystem's attachment plane. For example, for subsystem 0 – "spindle – fixture – workpiece" – it is reasonable to choose a point on the end surface of the spindle cut as the reference point. For subsystem 1 – "slide – tool" – it is reasonable to take the point of the slide's attachment surface, relative to which the cutting tool installed on it is referenced, as the reference point [13-16].

The point of application of the cutting forces is directly related to the arrangement scheme of the cutting tools in the setup, and, therefore, to the scheme for forming the manufactured dimensions.

Thus, the vector connecting the reference point of the subsystem and the point of application of the cutting forces, and, consequently, the matrix a_0, is primarily a characteristic of the setup implemented on the considered technological subsystem, rather than an inherent characteristic of the subsystem itself [17, 18].

The subsystem's own elastic properties in its comprehensive characteristic (1) are described by two basic elements: the matrices C and C_φ. Therefore, to form the comprehensive compliance characteristic of the subsystem, which can be used to calculate dimensional deviations for a given setup, it is sufficient to determine these basic characteristic elements: the compliance matrix e for translational (plane-parallel) displacements and the compliance matrix ξ for angular displacements.

To develop a method for determining these intrinsic matrices, we examine the physical meaning of each matrix element. As follows from analytical mechanics, compliance is the displacement resulting from the action of a unit force [2, 3, 9-11, 17-21].

Let us consider the meaning of the elements of the matrix e^0:

$$e^0 = \begin{pmatrix} e^0_{xx} & e^0_{xy} & e^0_{xz} \\ e^0_{yx} & e^0_{yy} & e^0_{yz} \\ e^0_{zx} & e^0_{zy} & e^0_{zz} \end{pmatrix}. \qquad (2)$$

The displacement of the subsystem, whose elastic properties are described by the matrix e^0, under the action of a force P (Px, Py, Pz) is described by the vector $\overline{r_0}$:

$$\overline{r_0} = e^0 \overline{P}; \qquad (3)$$

or in expanded form:

$$\begin{pmatrix} r_x \\ r_y \\ r_z \end{pmatrix} = \begin{pmatrix} e^0_{xx} & e^0_{xy} & e^0_{xz} \\ e^0_{yx} & e^0_{yy} & e^0_{yz} \\ e^0_{zx} & e^0_{zy} & e^0_{zz} \end{pmatrix} \begin{pmatrix} c_x t^{x_x} s^{y_x} v^{z_x} \\ c_y t^{x_y} s^{y_y} v^{z_y} \\ c_z t^{x_z} s^{y_z} v^{z_z} \end{pmatrix}. \qquad (4)$$

the projections of which onto the coordinate axes are defined as:

$$r_x = e^0_{xx} P_x + e^0_{xy} P_y + e^0_{xz} P_z ; \tag{5}$$

$$r_x = e^0_{xx} P_x + e^0_{xy} P_y + e^0_{xz} P_z ; \tag{6}$$

$$r_x = e^0_{xx} P_x + e^0_{xy} P_y + e^0_{xz} P_z . \tag{7}$$

As a rule, when studying the stiffness of machine tools and machine tool systems, the analysis is often limited to determining the so-called coordinate stiffnesses – j_x, j_y, j_z. By transitioning from stiffnesses to their inverse values – the compliances: $e_k = \frac{1}{j_k}$, k = x, y, z; we see that the specified parameters constitute the elements of the main diagonal of the compliance matrix (2):

$$\frac{1}{j_x} = e_{xx} ; \ \frac{1}{j_y} = e_{yy} ; \ \frac{1}{j_z} = e_{zz} . \tag{8}$$

The remaining elements of the compliance matrix (2) reflect the extent to which components of the cutting forces directed along other coordinate axes (not the one along which the coordinate displacement is considered) influence the magnitude of the coordinate displacement [22-30].

Schematic representation of the physical meaning of the compliance matrix elements can be expressed by equation (9). In it, the left arrow at each matrix element indicates the direction of displacement, while the right (double) arrow indicates the direction of action of the unit component of the cutting force. The corresponding matrix element characterizes the magnitude of the displacement in the specified direction due to the action of a unit force in the indicated direction.

$$\begin{pmatrix} \rightarrow e^0_{xx} \Rightarrow & \rightarrow e^0_{xy} \Uparrow & \rightarrow e^0_{xz} \otimes \\ \uparrow e^0_{yx} \Rightarrow & \uparrow e^0_{yy} \Uparrow & \uparrow e^0_{yz} \otimes \\ \otimes e^0_{zx} \Rightarrow & \otimes e_{zy} \Uparrow & \otimes e^0_{zz} \otimes \end{pmatrix} . \tag{9}$$

Such an understanding of the essence and physical meaning of the compliance matrix elements points to a fairly simple scheme for the practical determination of the matrix elements – it is sufficient to load the tested subsystem with a force in a given coordinate direction and measure the resulting displacements in all coordinate directions.

Experimental determination of the static matrix compliance characteristic. Understanding the essence and physical meaning of the compliance matrix elements indicates a possible practical scheme for determining these elements – it is enough to load the tested subsystem with a force in a given coordinate direction and measure the displacements in all coordinate directions [20-23].

We select three linearly independent force vectors (Fig. 1):

$$\begin{aligned} \overline{F_1} &= \left\{ F^1_x ; F^1_y ; F^1_z \right\} ; \\ \overline{F_2} &= \left\{ F^2_x ; F^2_y ; F^2_z \right\} ; \\ \overline{F_3} &= \left\{ F^3_x ; F^3_y ; F^3_z \right\} . \end{aligned} \tag{10}$$

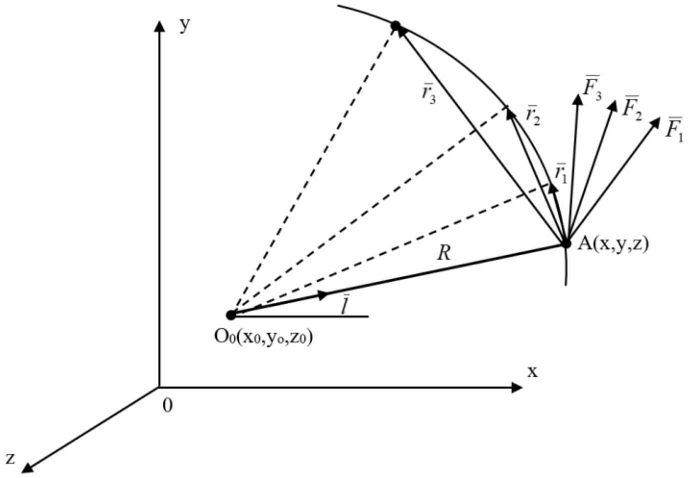

Fig. 1. *Scheme for determining the elements of the basic compliance matrices for translational (plane-parallel) displacements under static loading of the technological subsystem [23]*

From the projections of these vectors onto the coordinate axes, a force matrix can be formed:

$$F = \begin{pmatrix} F_x^1 & F_x^2 & F_x^3 \\ F_y^1 & F_y^2 & F_y^3 \\ F_z^1 & F_z^2 & F_z^3 \end{pmatrix}. \tag{11}$$

In an elastically deformable system, each of the force vectors (10) corresponds to a respective displacement vector:

$$\begin{aligned} \overline{r_1} &= \{r_x^1; r_y^1; r_z^1\}; \\ \overline{r_2} &= \{r_x^2; r_y^2; r_z^2\}; \\ \overline{r_3} &= \{r_x^3; r_y^3; r_z^3\}. \end{aligned} \tag{12}$$

For the set of displacement vectors, similarly to (11), a displacement matrix can also be formed:

$$r = \begin{pmatrix} r_x^1 & r_x^2 & r_x^3 \\ r_y^1 & r_y^2 & r_y^3 \\ r_z^1 & r_z^2 & r_z^3 \end{pmatrix}. \tag{13}$$

For the displacement vectors, the fundamental equation of analytical mechanics for linearly elastically deformable systems [21] holds:

$$\overline{r_i} = e\overline{F_i}; \ i = 1, 2, 3 \tag{14}$$

where e is the compliance matrix of the deformable system.

The system of displacement equations is expressed in matrix form and solved for the unknown compliance matrix by inverting the force matrix, allowing determination of all elements of the plane-parallel compliance matrix, with the same procedure applicable to the angular compliance matrix provided that both the applied force vectors and their points of application are chosen to ensure linear independence [20-23].

Three linearly independent moment vectors are formed and assembled into a moment matrix; the corresponding angular displacement vectors are likewise arranged into a displacement matrix, and, based on the fundamental equations of linear elastic systems, these quantities are related through the angular compliance matrix, which fully characterizes the rotational deformation behavior of the subsystem.

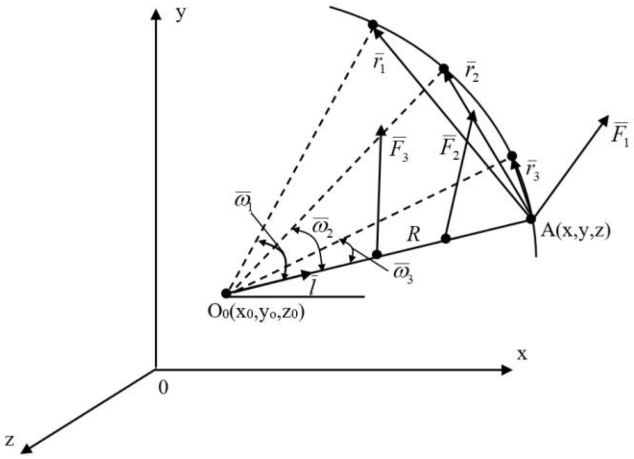

Fig. 2. *Scheme for determining the elements of the basic compliance matrices for angular displacements under static loading of the technological subsystem [23]*

In the practical implementation of the developed method for determining the compliance matrices, it is sufficient to use a single set of three forces, from which the three linearly independent moments were generated [21-25].

Experimental determination of the dynamic compliance characteristic. The most effective means of constructing the dynamic characteristic of a machine tool at present is modal analysis [21].

Modal analysis determines the dynamic characteristics of a metal-cutting machine by exciting its structure within a specified frequency range and recording vibration responses using sensors and specialized software. Excitation methods include impulse loading, narrowband noise, and sweep sine signals. Although a real machine, as a distributed-parameter system, theoretically possesses infinitely many natural frequencies, in practice only a finite number of low-frequency modes relevant to cutting stability and resonance zones are identified [22, 28].

Dynamic compliance evaluation requires complex and costly equipment and is therefore mainly applied during machine certification. In production conditions, operational assessment is carried out using the production method of stiffness evaluation, based on machining under standardized conditions and estimating compliance from residual surface steps. In its conventional form, this method determines the overall compliance of the technological system in the direction of the machined dimension, primarily along radial and axial axes, corresponding to individual elements of the combined compliance matrix [30].

For matrix-based machining accuracy models, subsystem-level compliance characteristics are required. Modern CNC machines enable high-precision definition of setup geometry and in-process measurement of machined surface positions. Combined with cutting force models, this allows determination of displacement vector components during machining and subsequent identification of all elements of the subsystem compliance matrix. Considering the influence of random errors, statistical evaluation is recommended.

The proposed approach enables direct determination of matrix dynamic compliance characteristics on CNC machines equipped with integrated measurement and diagnostic systems. On this basis, standardized control programs can be developed for each setup type, forming a built-in diagnostic module of the machine tool system.

Conclusions

1.The concept of the comprehensive compliance characteristic of a technological system has been introduced. This characteristic encompasses not only the subsystem's inherent elastic properties but also the parameters of the specific setup for which the compliance is evaluated.

2.It has been demonstrated that determining two fundamental characteristic elements is sufficient to formulate the comprehensive compliance characteristic of a subsystem. This formulation enables the calculation of deviations in machined dimensions for a given setup.

3.An experimental methodology for determining the static matrix compliance characteristic has been developed, alongside a statistical evaluation framework for the comprehensive compliance characteristic.

4.Based on the proposed method for determining the dynamic compliance of a technological subsystem, standard control programs can be developed for CNC machine systems corresponding to each setup type. These standard programs are intended to serve as integral components of machine-specific diagnostic modules.

REFERENCES

[1]. Bogatenkov S.A., Sazonova N.S., Guzeev, V.I., Yusubov N.D., Abbasova G.M. *Increasing the Productivity of Multitool Machining on Automated Lathes by Optimizing the Tool Positions*. Russian Engineering Research, 2021. Vol. 41, pp.1075-1079.

[2]. Yusubov N.D. *Fundamentals of matrix theory of accuracy of multi-tool turning. (Principles and structure of the theory, design and management of multi-tool machining processes)*. AV Akademikerverlag Gmbh & Co. KG / LAP LAMBERT Academic Publishing, Saarbrücken, 2013.

[3]. Yusubov, N.D. *Multi - tool machining on automatic lathes (Matrix theory of multi-tool machining accuracy on modern CNC lathes)*. AV Akademikerverlag Gmbh & Co. KG / LAP LAMBERT Academic Publishing, Saarbrücken, 2013.

[4]. Rasulov N.M., Nadirov U.M., Alekberov M.Z. *Generalized Assessment of Machined Surfaces Quality. Russian Engineering Research*, 2020. Vol. 40, pp. 822-825.

[5]. Bogatenkov S.A. et. al. *Decision-making in the conditions of introduction of automated design systems of technological processes*. Socar Proceedings, 2022. Special İssue 1, pp. 006-010.

[6]. Aziz S.Sh. (2020). Theoretical studies of the dynamic characteristics of the internal lapping process. *Vestnik of Nosov Magnitogorsk State Technical University*, 18(2), 30–37.

[7]. Yusubov N., Abbasova H., Dadashov R. *Theoretical basis for the development of an algorithmic unified complex of mathematical models of cutting forces*. Machine science, 2023. N1, pp. 55-60.

[8]. Yusubov N., Abbasova H., Khankishiyev İ. *Development of a planning theory for multi-tool machining with the possibilities of modern CNC machine tools*. Forschung in Ingenieurwesen, 2021. Vol. 85, pp.661-678. https://doi.org/10.1007/s10010- 021-00478-7

[9]. Yusubov N.D. *Matrix models of the accuracy in multitool two-support setup.* Russian Engineering Research, 2009. Vol. 29, pp. 268-271.

[10]. Yusubov N., Khankishiyev İ., Abbasova H., Mammadov E., Huseynov R. *Matrix models of machining errors in multi-tool multi-carriage adjustments*. International Journal on "Technical and Physical Problems of Engineering" (IJTPE), 2023. Vol.15, issue 56, pp. 309- 315.

[11]. Bogatenkov S.A., Yusubov N.D. *Planning of personal trajectors of development: systems of automated design*. Bulletin of the South Ural State University. Series: Computer Technologies, Control, Radio Electronics, 2019. Vol.19, issue 1, pp. 139-145.

[12]. Aziz S.Sh. *Kinematic features of the lapping process and determination of its basic parameters*.

Computational Nanotechnology, 2020, Vol. 7, No. 3, pp. 11–16.

[13]. Rasulov, N. M., Nadirov, U. M., Abbasova, I. A., Mammadov, A. S., Shabiyev, E. T. *Advanced technology for machining tapered threads of pipe coupling*. American Journal of Engineering and Technology Management, 2020, 5(2), pp.35–40.

[14]. Bashirov, R. J., Amirov, F. G. *Method for determining the thermal state of the cylinder sleeve during centrifugal induction sintering*. Izvestiya vysshikh uchebnykh zavedeniy. Mashinostroenie, 2022, pp.33–41.

[15]. Amirov, F. G. (2020). *Some features of increasing the productivity of automatic lines*. Proceedings of Higher Educational Institutions, Mechanical Engineering, (9), 18–23.

[16]. Yusubov N., *Experimental determination of the cutting force in turning*. Mechanical Engineering, 2003. Vol. 43, issue 2, pp. 23-25.

[17]. Yusubov N.D., Movlazade V.Z., Abbasova H.M. *Accuracy models of machining in multi-tool adjustments*. Proceedings of the 8th International Conference on Control and Optimization with Industrial Applications (COIA 2022), 2022. Vol. 2, pp. 477-479.

[18]. Yusubov N.D., Movlazade V.Z., Abbasova, H.M. *Research of sensitivity of full-factor models of scattering fields of dimensions performed in multi-tool machining*. The 8th International Conference on Control and Optimization with Industrial Applications (COİA 2022), vol. II, pp. 480-482.

[19]. Koshin A.A., Yusubov N.D. *Elements of the matrix theory of multi-tool machining accuracy in spatial adjustments*. Technical journal "Bulletin of mechanical engineering", 2013, 9, pp. 13-17.

[20]. Yusubov N.D., Abbasova H.M. *Full-factor matrix model of accuracy of dimensions performed on multi-purpose CNC machines*. Obrabotka metallov (tekhnologiya, oborudovanie, instrumenty) = Metal Working and Material Science, 2021, vol. 23, no. 4, pp. 6–20.

[21]. Yusubov N.D., Abbasova H.M. *Models for Machining Accuracy in Multi-Tool Adjustment*. International Journal of Automotive and Mechanical Engineering, 2020, vol. 17, no. 3, pp. 8067-8085.

[22]. Yusubov N.D., Abbasova H.M. *Full Factorial Models of Dimensional Accuracy of MultiTool Machining on Automatic Turning Machines*. Bulletin of the South Ural State University. Ser. Mechanical Engineering Industry, 2019, vol. 19, no. 1, pp. 56–67.

[23]. Yusubov N., Abbasova H. *Model of machining process control on multitool single-carriage adjustments*. Machine science, 2023. N1, pp. 22-27.

[24]. Yusubov N., Abbasova H., Dadashov R., Salmanov I. *On the matrix generalization of the theory of machining accuracy*. Machine Science, 2022, Vol. 11, No. 2, pp. 23–36.

[25]. Ardashev D.V., Yusubov N., Shipulin L.V., Degtyareva-Kashutina A. *Research on chrome plating quality of the internal surfaces in the hydraulic drives with hydrostatic guideways*. Materials Science Forum, 2021, Vol. 1037, pp. 417–422.

[26]. Yusubov N.D. *Matrix models of processing accuracy in multitool turning*. Mechanical Engineering Technology, 2013, No. 1, pp. 57–63.

[27]. Koshin A.A. *Obrabotka na tokarnykh stankakh: naladka, rezhimy rezaniya. Spravochnik* (Processing on Turning Machines: Adjustment and Cutting Mode: Handbook). Siti-Print, 2012.

[28]. Yusubov N.D. *Algorithmization of analytical model of dimensions stray field, executed in multi-tool multi-carriage adjustments*. Vestnik Mashinostroeniya, 2008, No. 2, pp. 54–56.

[29]. Yusubov N.D. *Matrix models of working accuracy at single-cutter turning*. Journal Tekhnologiya Mashinostroeniya, 2009, No. 1, pp. 41–45.

[30]. Yusubov, N., & Abbasova, H. *Control of processing at multi-tool two-support setup*. Vestnik Mashinostroyeniya, 2020, No. 3, pp.67–73.

Received: 15.08.2023
Accepted: 26.01.2024